Theory Appendix for "Can Enforcement Backfire? Crime Displacement in the Context of Customs Reform in the Philippines."

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I develop here a fuller version of the model of smuggling displacement from Section 2 of the paper "Can Enforcement Backfire? Crime Displacement in the Context of Customs Reform in the Philippines." I assume a general form for the convex cost of smuggling, and account for the endogeneity of the volume imported by each firm. The basic conclusion—that a targeted increase in enforcement can raise or lower the smuggling rate—does not change.

1 Basic framework

Many identical importers in perfect competition import a standard good for sale in the domestic market. There is free entry and exit from the import market. Each importer chooses an amount of imports M. An importer's costs include its costs of obtaining the good from the foreign supplier, various costs of domestic marketing and distribution, a fixed cost of operating the firm, and import duties levied at an ad valorem tariff rate τ. To avoid tariffs, an importer can smuggle a fraction of its imports, which entails additional fixed and variable costs.

An importer chooses between two smuggling methods, method 1 and method 2. S_j is the amount of imports that the importer hides from customs authorities using smuggling method j. The following function denotes an importer's total profit:

$$PM - M - c(M) - E - \tau M \tag{1}$$
$$+\tau \left(S_1 + S_2\right) - f\left(S_1, v_1\right) - f\left(S_2, v_2\right) - f\left(S_1 + S_2, g\right)$$
$$-F_1 \cdot \mathbf{1}\left(S_1 > 0\right) - F_2 \cdot \mathbf{1}\left(S_2 > 0\right)$$

The first four terms of this expression are the importer's costs of making legitimate sales of imports in the domestic market. PM is gross revenues from selling its imports at the domestic market price P. Let the domestic market be small with respect to the world market, so that the landed cost (price paid to the overseas supplier plus freight costs) of each unit of imports can be assumed exogenous. Normalize this landed cost per unit of imports to one, so that M is the total landed cost. $c(M)$ is the importer's domestic costs of marketing and distribution, assumed to be convex in the amount of imports ($\frac{\partial c}{\partial M} > 0$, $\frac{\partial^2 c}{\partial M^2} > 0$). Let E be the importer's fixed cost of importing a nonzero amount. τM is tariffs payable on total imports M.

The remaining terms of expression (1) are the importer's net benefit from smuggling. The importer's choice of S_1 and S_2 yields savings of $\tau(S_1 + S_2)$ due to evaded tariffs.

Variable smuggling costs are represented by the next three terms. Let $f(a, b)$ be an increasing function, where a is an amount of smuggling and b is a parameter. Let $f(a, b)$ be convex in a ($\frac{\partial f(a,b)}{\partial a} > 0$, $\frac{\partial^2 f(a,b)}{\partial a^2} > 0$), and let variable costs be zero when smuggling is zero ($f(0, b) = 0$). The parameters v_1, v_2, and g characterize the extent to which variable costs rise in the relevant amounts of smuggling. Higher values of these parameters indicate that marginal costs of smuggling are higher for every amount smuggled ($\frac{\partial^2 f(a,b)}{\partial a \partial b} > 0$).

Some costs of smuggling should depend on the overall amount of smuggling, regardless of smuggling method; such costs are captured in the term $f(S_1 + S_2, g)$.[1] For example, the cost of keeping alternate sets of accounting records (honest ones solely for internal use versus dishonest ones for the eyes of government auditors) should rise in the total amount smuggled. In addition, bribes for high-level government officials could also be increasing in the total amount smuggled, independent of the type of smuggling.

The last two terms of equation (1) are fixed costs of smuggling. $\mathbf{1}(S_j > 0)$ is an indicator function, taking the value of 1 if $S_j > 0$ and zero otherwise. If the importer chooses to do some nonzero amount of smuggling via method j, it bears a positive cost F_j.

I restrict attention to the case where the importer chooses to use either smuggling method one or two, but not both simultaneously.[2] Because I am interested in modeling smuggling, I also impose that the fixed costs of either smuggling method not be too large as to discourage

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[1]Costs convex in the total amount of smuggling, in combination with the fixed costs, make it possible for the importer not to choose to use both smuggling methods simultaneously. If costs did not rise in the overall smuggling rate specifically (if the term $f(S_1 + S_2, g)$ did not exist), the importer would always use both smuggling methods as long as each was profitable on its own. Intuitively, in the absence of costs convex in total smuggling ($S_1 + S_2$), the importer would have a much stronger incentive to avoid convex costs that rise in each smuggling method separately by spreading its smuggling activity across both methods.

[2]It is easy to show that if the importer chooses to use both smuggling methods simultaneously, an increase in any of the variable cost parameters must reduce the amount of smuggling.

smuggling entirely. (In other words, I study the case where the importer chooses to use exactly one smuggling method both before and after the increase in enforcement.[3])

When the importer uses just smuggling method j, the importer's total profit (denoted π_j) is:

$$\pi_j \equiv PM - M - c(M) - E - \tau M + \tau S_j - f(S_j, v_j) - f(S_j, g) - F_j \tag{2}$$

For each smuggling method j, denote the profit-maximizing amounts of imports and smuggling as M^* and S_j^*. (P will also be determined endogenously, but I will not solve for it explicitly.)

M^* and S_j^* are determined as follows. Perfect competition and free entry and exit imply that price is driven to average total cost (the zero-profit condition), while firm-level maximization requires that price equals marginal cost. Therefore, in equilibrium M^* and S_j^* must satisfy the condition that average total cost equals marginal cost:

$$\frac{M^* + c(M^*) + E + \tau M^* - \tau S_j^* + f(S_j^*, v_j) + f(S_j^*, g) + F_j}{M^*}$$
$$= 1 + \frac{\partial c(M^*)}{\partial M} + \tau \tag{3}$$

In addition, each importer's chosen amount of smuggling S_j^* must maximize expression (2), so that the marginal benefit of an additional unit of smuggling (τ) will equal the marginal cost of that smuggled unit:

$$\tau = \frac{\partial f(S_j^*, v_1)}{\partial S_j^*} + \frac{\partial f(S_j^*, g)}{\partial S_j^*}. \tag{4}$$

Of course, it must also be true that the amount smuggled is less than or equal to the amount imported ($S_j^* \in [0, M^*]$). I assume an interior optimum in all that follows.

Let B_j be the net benefit from smuggling when using smuggling method j and choosing the optimal smuggling amount (tariff savings minus variable and fixed costs):

$$B_j \equiv \tau S_j^* - f(S_j^*, v_j) - f(S_j^*, g) - F_j \tag{5}$$

In equilibrium, importers should not have an incentive to change smuggling methods, so each

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[3]Without loss of generality, consider the choice between using method 1 alone and using both methods. Let S_1^* be the optimal amount of smuggling if the importer only uses method one. Let S_1^{**} and S_2^{**} be the optimal amounts of the two smuggling methods if the importer must use both methods. The importer will prefer method one alone to using both methods if:

$$\tau S_1^* - f(S_1^*, v_1) - f(S_1^*, g) > \tau(S_1^{**} + S_2^{**}) - f(S_1^{**}, v_1) - f(S_2^{**}, v_2) - f(S_1^{**} + S_2^{**}, g) - F_2.$$

This expression can hold for large enough fixed costs of using method two, F_2.

importer will choose the same method: the one yielding the highest net benefit from smuggling. Formally, importers choose smuggling method j such that $B_j > B_k$, for $j \neq k$. Given the common choice j, the solution to equations (3) and (4) yields the equilibrium amounts imported and smuggled, M^* and S_j^*.

2 Impact of increased enforcement on one smuggling method

Without loss of generality, let the initially chosen smuggling method be method 1. A customs reform (like PSI) targeting method 1 is implemented, raising its variable cost parameter v_1. The importer can choose to continue using smuggling method 1, or switch to using smuggling method 2 if smuggling method 1 no longer allows the maximum net benefit between the two smuggling methods.

The *smuggling rate* is the ratio of the amount of smuggling to total imports in equilibrium:

$$
\gamma = \begin{cases}
\frac{S_1^*}{M^*}, & \text{if the importer uses smuggling method 1} \\
\frac{S_2^*}{M^*}, & \text{if the importer uses smuggling method 2}
\end{cases}
$$

The change in the smuggling rate when v_1 increases will depend on whether the importer continues using smuggling method 1 or switches to method 2, as described in the following two claims.

Claim 1 *An increase in v_1 always leads to a lower smuggling rate if the importer continues to use smuggling method 1.*

Proof. The change in the smuggling rate can be expressed as follows (using the Chain Rule):

$$
\frac{d\gamma}{dv_1} = \frac{1}{M^*}\left(\frac{\partial S_1^*}{\partial v_1} - \gamma \frac{\partial M^*}{\partial v_1}\right) \tag{6}
$$

Totally differentiate condition (4) with respect to v_1 and rearrange to obtain:

$$
\frac{\partial S_1^*}{\partial v_1} = -\frac{\partial^2 f\left(S_1^*, v_1\right)}{\partial S_1^* \partial v_1} \frac{1}{\frac{\partial^2 f\left(S_1^*, v_1\right)}{\partial S_1^{*2}} + \frac{\partial^2 f\left(S_1^*, g\right)}{\partial S_1^{*2}}}
$$

Because $\frac{\partial^2 f(S_1, v_1)}{\partial S_1 \partial v_1}$, $\frac{\partial^2 f(S_1, v_1)}{\partial S_1^2}$, and $\frac{\partial^2 f(S_1, g)}{\partial S_1^2}$ are all positive, $\frac{\partial S_1^*}{\partial v_1} < 0$.

To determine the sign of $\frac{\partial M^*}{\partial v_1}$, totally differentiate equation (3) with respect to v_1 and rearrange

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to obtain

$$\frac{\partial M^*}{\partial v_1} = \frac{1}{c'' M^*} \left(\frac{\partial f\left(S_1^*, v_1\right)}{\partial v_1} + \frac{\partial S_1^*}{\partial v_1} \left(\frac{\partial f\left(S_1^*, v_1\right)}{\partial S_1^*} + \frac{\partial f\left(S_1^*, g\right)}{\partial S_1^*} \right) - \tau \frac{\partial S_1^*}{\partial v_1} \right).$$

Substitute equation (4) in this expression to obtain

$$\frac{\partial M^*}{\partial v_1} = \frac{1}{c'' M^*} \frac{\partial f\left(S_1^*, v_1\right)}{\partial v_1}.$$

This expression is greater than zero because $c'' > 0$ and $\frac{\partial f\left(S_1^*, v_1\right)}{\partial v_1} > 0$ by assumption. So $\frac{d\gamma}{dv_1} < 0$ by equation (6). ∎

The intuition behind this result is straightforward: a rise in the marginal cost of smuggling leads the importer to reduce the amount smuggled. The domestic price of imports rises, leading each importer to import more. The amount smuggled as a share of total imports for each importer falls unambiguously.

Claim 2 *An increase in v_1 has an ambiguous impact on the smuggling rate if the importer switches to using smuggling method 2.*

Proof. For each importer after the switch to smuggling method 2, let the new amounts of smuggling and imports be $S_2^* \equiv S_1^* + \kappa$ and $M^{*\prime} \equiv M^* + \mu$, respectively. The change in the smuggling rate is

$$\begin{aligned} \Delta\gamma &= \frac{S_2^*}{M^{*\prime}} - \frac{S_1^*}{M^*} = \frac{S_1^* + \kappa}{M^* + \mu} - \frac{S_1^*}{M^*} \\ &= \frac{M^*\kappa - S_1^*\mu}{\left(M^* + \mu\right) M^*} \end{aligned}$$

The denominator of this expression is positive, so the sign of $\Delta\gamma$ will be determined by the sign of the numerator, $M^*\kappa - S_1^*\mu$.

Rewrite equation (3) to obtain

$$\tau S_j^* - f\left(S_j^*, v_j\right) - f\left(S_j^*, g\right) - F_j = c(M^*) - M^*\frac{\partial c(M^*)}{\partial M} + E \tag{7}$$

Note that the left-hand-side of this expression is B_j, the net benefit from smuggling. If an importer initially chose smuggling method 1, it must have been true that $B_1 > B_2$. Switching from smuggling method 1 to method 2 (with no change in method 2's cost parameters) therefore

must involve a decline in the net benefit from smuggling, so that the left-hand-side of equation (7) falls.

For equation (7) to continue to hold once the importer has switched to method 2, the right-hand-side must also fall. Differentiating the right-hand-side of equation (7) with respect to M yields $-M^* \frac{\partial^2 c(M^*)}{\partial M^2}$, which is less than zero because $c(\cdot)$ is a convex function.

An increase in M causes the right-hand-side of equation (7) to fall. $M^{*\prime}$ must therefore be higher than M^* for equation (7) to hold after the switch to method 2. So $\mu > 0$.

κ can be either positive or negative. Equation (4) implies that if $v_2 > v_1$ then $\kappa < 0$, and if $v_2 < v_1$ then $\kappa > 0$. If $\kappa < 0$, then $\Delta\gamma < 0$.

But if $\kappa > 0$, the sign of $(M^*\kappa - S_1^*\mu)$ is indeterminate. Therefore the sign of $\Delta\gamma$ is also indeterminate. ■

It is possible for an importer to smuggle more after the switch if smuggling method 2 has lower variable costs than method 1 ($v_2 < v_1$). To explain why it was not initially chosen over method 1, method 2's fixed costs should be enough larger than those of method 1 ($F_2 > F_1$). The domestic market price of imports will rise because smuggling has become more expensive, and each importer will import more as a result (because price equals marginal cost, and costs are convex.) But as long as the increase in an importer's average total costs is not too large (so that the increase in imports supplied by each importer is not too large), it is possible for the smuggling rate to rise.[4]

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[4]We could also think of the customs reform raising the fixed cost F_1 instead of the variable cost parameter v_1. In this case, there is no change in the smuggling rate if the importer still chooses to smuggle using method 1 (the fixed cost parameter does not affect the optimal amount of smuggling). However, the impact on the smuggling rate is still ambiguous if the importer switches to smuggling method 2; the reason for the switch is immaterial.